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A STUDY OF CONGRUENCES ON ADDITIVELY REGULAR NEAR-Γ-SEMIRINGS

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Abstract : To enrich the interplay between a near-Γ-semiring and its operator seminearrings which have been started in (Sardar and Mukherjee, 2014), in this paper at first an order isomorphism between the poset of all Γ-near-ring congruences on a near-Γ-semiring and the poset of all of its normal full k-ideals has been established. Using that result together with the fact that there exists an order isomorphism between the poset of all normal full k-ideals of a near-Γ-semiring and that of its left (right) operator seminearring, an order isomorphism between the poset of all Γ-near-ring congruences on a near-Γ-semiring and the poset of all near-ring congruences on its left (right) operator seminearring has been obtained here so that the complexity of a Γ-near-ring congruence on a near-Γ-semiring can be reduced by viewing it through a normal full k-ideal of its left (right) operator seminearring.

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Key Words and Phrases: seminearring; near-Γ-semiring; near-ring congruence; Γ-near-ring congruence; normal full k-ideal

I. Introduction. In (Dutta and Sardar, 2003), Dutta and Sardar associated two semirings, namely the operator semirings, with a Γ-semiring S and observed a nice interplay between a Γ-semiring and its operator semirings which plays an important role in developing the theory of Γ-semirings as well as of semirings. Taking impetus from this work, in (Sardar and Mukherjee, 2014), Sardar et al. introduced the notion of operator seminearrings of a near-Γ-semiring and then studied the interplay between them as was done for Γ-semirings. In the current paper, our aim is to extend the interrelationships studied in (Sardar and Mukherjee, 2014) and study Γ-near-ring congruences on a near-Γ-semiring via near-ring congruences on its operator seminearrings.

Now we give a glimpse of motivation and development of the topic under consideration. It is well-known that $End(S)$, the set of all endomorphisms of a commutative semigroup $(S, +)$ forms a semiring (Golan, 1999). But instead of $End(S)$

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